Effective Theory of Wilson Lines and Deconfinement

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(Dated: September 21, 2006)

To study the deconfining phase transition at nonzero temperature, I outline the perturbative construction of an effective theory for straight, thermal Wilson lines. Certain large, time dependent gauge transformations play a central role. They imply the existence of $U(1)$ interfaces, which can be used to determine the form of the effective theory as a gauged, nonlinear sigma model of adjoint matrices. Especially near the transition, the Wilson line may undergo a Higgs effect. As an adjoint field, this can generate eigenvalue repulsion in the effective theory.

Recent results at the Relativistic Heavy Ion Collider (RHIC) demonstrate qualitatively new behavior for the collisions of heavy ions at high energies [1]. RHIC appears to have entered a region above T_c, the temperature for deconfinement, reaching up to temperatures a few times T_c. The experimental results cannot be explained if the transition is directly from a confined phase to a perturbative Quark-Gluon Plasma (QGP). Instead, RHIC seems to probe a novel region, which has been dubbed the "sQGP" [2].

In this paper I sketch how to develop an effective theory for the sQGP. Classically, the model is a familiar spin system, a gauged principal chiral field [3]; beyond leading order, it is more general. A mean field approximation to the effective theory gives a random matrix model [3]. Such models are dominated by eigenvalue repulsion from the Vandermonde determinant in the measure. For a $SU(\infty)$ gauge theory in a small volume, deconfinement is driven by exactly such a mechanism [4]. I indicate later how eigenvalue repulsion might arise in infinite volume, from the Higgs effect for an adjoint matrix.

By the converse of asymptotic freedom, the running QCD coupling, $\alpha_s(T) = g^2(T)/(4\pi)$, increases as the temperature decreases. Thus a natural possibility is that in the sQGP, $\alpha_s(T)$ becomes very large as the temperature $T \to T_c^+$. For the phenomenology of a strongly coupled, deconfined phase, see [2].

A definitive value for $\alpha_s(T)$ can be obtained by matching correlation functions, for the original theory in four dimensions, with an effective theory in three dimensions [5, 6, 7, 8, 9]:

$$\begin{aligned} \mathcal{L}^{eff}_{small\ A_0}(A_i, A_0) \;&=\; \frac{1}{2}\,\mathrm{tr}\,G_{ij}^2 \;+\; \mathrm{tr}\,|D_i A_0|^2 \\ &+ m_D^2\,\mathrm{tr}\,A_0^2 \;+\; \kappa_1\left(\mathrm{tr}\,A_0^2\right)^2 \;+\; \kappa_2\,\mathrm{tr}\,A_0^4\,. \end{aligned} \tag{1}$$

This is the Lagrangian for a massive, adjoint scalar field, A_0, coupled to static magnetic fields, A_i: A_0 and A_i are the time like and space like components of the vector potential, G_{ij} is the non-abelian magnetic field strength, and D_i the covariant derivative. Fields and couplings are normalized as in four dimensions, with the three dimensional action $\int d^3x/T$ times the Lagrangian. At leading order, integrating out the four dimensional modes produces a Debye mass for A_0, $m_D^2/T^2 \sim \alpha_s$, and quartic couplings, κ_1 and κ_2, $\sim \alpha_s^2$, with each a power series in α_s.

This effective theory represents an optimal resummation of perturbation theory. As such, it applies only when fluctuations in A_0 are small. Computing the pressure to four loop order, $\sim \alpha_s^3$, the results are complete up to one undetermined constant [9]. Even with the most favorable choice for this constant, however, the pressure does not agree with that from numerical simulations on the lattice below temperatures of $\sim 3T_c$ [6, 7].

These computations are done in imaginary time, where the "energies" are multiples of $2\pi T$. Thus the coupling constant $\alpha_s(T)$ runs with a scale which is of order $\sim 2\pi T$ [5]. Computations to two loop order show that even better, this mass scale is $\sim 9T$ in QCD [7]. For $T_c \sim 175$ MeV, this is ~ 1.6 GeV; at $3T_c$, it is ~ 4.7 GeV. While these mass scales are not asymptotic, neither are they obviously in a non-perturbative regime: *e.g.*, $\alpha_s(1.6\text{ GeV}) \sim 0.28$ [7]. Hence the question becomes: why does this effective theory fail between T_c and $\sim 3T_c$, if the coupling is *not* that large?

To see how this might occur, consider a straight, thermal Wilson line in the fundamental representation:

$$L(x,\tau) = P\, e^{\,ig\int_0^\tau A_0(x,\tau')\,d\tau'}\;; \tag{2}$$

P denotes path ordering, x is the spatial position, and τ, the imaginary time, runs from 0 to $1/T$. A closed loop is formed by wrapping all of the way around in imaginary time, $L(x, 1/T)$. As this quantity arises frequently, I denote it by $L(x)$.

The Wilson line is a matrix in color space, and so is not directly gauge invariant: under a gauge transformation $\mathcal{U}(x,\tau)$, $L(x) \to \mathcal{U}^\dagger(x, 1/T)L(x)\,\mathcal{U}(x, 0)$. The trace of the Wilson line is gauge invariant, and is the Polyakov loop in the fundamental representation. Normalizing so that this loop is one when $A_0 = 0$, then its expectation value should be near one if $gA_0/(2\pi T)$ is small. Numerical simulations of a lattice $SU(3)$ gauge theory show that while the expectation value of the renormalized triplet loop is near one at $3T_c$, this is not so when $T < 3T_c$. Without dynamical quarks, it drops to a value of ≈ 0.45 at T_c [10, 11, 12, 13]; its value with dynamical quarks is similar [14].

Since the triplet loop is significantly less than one between T_c and $\sim 3T_c$, in this region it is necessary to extend the program of [5, 6, 7, 8, 9] to construct an effective, three dimensional theory for arbitrary values of $gA_0/(2\pi T)$. While A_0 can be large, as it applies only for distances $\gg 1/T$, we can assume that all spatial momenta are small relative to $2\pi T$ [15, 16, 17, 18, 19, 20]. This is like chiral perturbation theory, with temperature playing the role of the pion decay constant.

Certainly the effective theory must be invariant under static gauge transformations, $\mathcal{U}(x,\tau) = \mathcal{U}(x)$. In addition, and somewhat unexpectedly for a theory in three dimensions, certain time dependent gauge transformations matter. For a $SU(N)$ gauge group, consider

$$\mathcal{U}_c(\tau) = e^{2\pi i\, \tau T\, t_N} \ , \ t_N = \begin{pmatrix} 1_{N-1} & 0 \\ 0 & -(N-1) \end{pmatrix} \ ; \quad (3)$$

1_{N-1} is the unit matrix. This is spatially constant and strictly periodic in τ, $\mathcal{U}_c(1/T) = e^{2\pi i}\, 1_N = \mathcal{U}_c(0)$; thus it appears rather trivial. Instead, it turns out to be essential in constraining the form of the effective Lagrangian at large A_0. Since they don't alter the boundary conditions in imaginary time, similar gauge transformations exist for any gauge group, coupled to matter fields in arbitrary representations.

In four dimensions, the electric field is a sum of two terms, $D_i A_0 - \partial_0 A_i$. Under (3), diagonal elements of A_0 are shifted by a constant amount, $A_0^{diag} \to A_0^{diag} + 2\pi T t_N/g$, while off-diagonal elements of A_0 and A_i undergo time dependent rotations. Thus the first term, $D_i A_0 = \partial_i A_0 - ig[A_i, A_0]$, changes if $[A_i, t_N] \neq 0$. This is unavoidable for some off-diagonal components of A_i. Since the complete electric field transforms homogeneously under gauge transformations, the piece $\sim [A_i, t_N]$ is cancelled by the time dependent rotation of A_i in the second term, $-\partial_0 A_i$.

This also shows that it is not apparent how to implement these gauge transformations in an effective theory. The simplest thing is just to drop all time derivatives, taking the effective electric field $E_i = D_i A_0$. This is not invariant, however, under the shift in A_0^{diag}; and with no time derivatives, there is nothing to cancel the piece $\sim [A_i, t_N]$. Since the shift in A_0 is $\sim T/g$, this can be ignored at small A_0, such as for (2).

The problem cannot be ignored at large A_0, and arose previously. The N^{th} root of $\mathcal{U}_c$ is an aperiodic gauge transformation, $= e^{2\pi i/N} 1_N$ at $\tau = 1/T$. If there are no dynamical quarks present, this is an allowed gauge transformation, and reflects the $Z(N)$ center symmetry of a $SU(N)$ gauge group [21]. Ref. [19, 20] computed in the presence of nonzero, background fields for both A_0 *and* A_i, and showed that if the effective Lagrangian is formed from terms such as $D_i A_0$, then at one loop order, the $Z(N)$ center symmetry appears to be violated. The argument above, applied to $\mathcal{U}_c^{1/N}$, shows that even classically, $E_i = D_i A_0$ is not consistent with the requisite $Z(N)$ symmetry [19].

The significance of these large gauge transformations can be understood by looking at the Wilson line. Since L is a $SU(N)$ matrix, $L^\dagger(x) L(x) = 1_N$, it can be diagonalized by a unitary transformation,

$$L(x) = \Omega(x)^\dagger\, e^{i\lambda(x)}\, \Omega(x) \ . \quad (4)$$

$\lambda(x)$ is a diagonal matrix, with elements λ_a, $a = 1 \ldots N$. As $\det(L) = 1$, $\mathrm{tr}\, \lambda(x) = 0$, modulo 2π. Under static gauge transformations, $\mathcal{U}(x) = \mathcal{U}$, the adjoint covariant derivative and the Wilson line transform similarly, $D_i \to \mathcal{U}^\dagger D_i \mathcal{U}$ and $L \to \mathcal{U}^\dagger L \mathcal{U}$. Hence the λ_a do not change, while Ω is gauge dependent, $\Omega \to \Omega\, \mathcal{U}$.

The λ_a can change under time dependent gauge transformations: under (3), $\lambda \to \lambda + 2\pi t_N$, so each λ_a shifts by an integral multiple of 2π. Thus gauge transformations such as (3) ensure that the λ_a's are periodic variables. Of course this is obvious from the definition of the Wilson line, since its eigenvalues are $e^{i\lambda_a}$.

This periodicity is present for an abelian gauge group, where the Wilson line is just a phase, $L = e^{i\lambda}$. Shifting $\lambda \to \lambda + 2\pi$ is an Aharonov-Bohm effect, where the Wilson line, in imaginary time, wraps around a patch of magnetic flux in a fictitious fifth dimension. For the partition function, this represents a global gauge rotation upon states [22].

The periodicity illustrates elementary topology. At nonzero temperature, imaginary time is isomorphic to a sphere in one dimension, S^1. Topologically nontrivial windings are given by mappings from S^1 into $U(1)$, and are classified by the first homotopy group, $\pi_1(U(1)) = \mathcal{Z}$, where $\mathcal{Z}$ is the group of the integers.

The result for a nonabelian group is an exercise in abelian projection for the Wilson line [23]. If there are r diagonal generators in the Cartan subalgebra of the gauge group, they define an abelian subgroup of $U(1)^r$, the direct product of r $U(1)$'s. Nontrivial windings are then given by $\pi_1(U(1)^r) = \mathcal{Z}^r$. In $SU(N)$, $r = N-1$, and t_N is one of these diagonal generators. (Note that the Wilson line in (4) is also invariant under a hidden symmetry of $U(1)^r$ on the left, $\Omega(x) \to e^{i\theta(x)}\, \Omega(x)$; $\theta(x)$ is a diagonal matrix, $\mathrm{tr}\, \theta(x) = 0$, modulo 2π.)

The effective Lagrangian must respect the periodicity of the λ_a's. This is automatic if it is constructed from the Wilson line. What is then obscure is the form of the effective electric field, E_i. Consider

$$E_i(x) = \frac{T}{ig}\, L^\dagger(x) D_i L(x) \ . \quad (5)$$

Like the original electric field, this is gauge covariant, $E_i \to \mathcal{U}^\dagger E_i \mathcal{U}$. It is also hermitean, and so is not $\sim D_i L$. If the gauge group has a center symmetry, then E_i is trivially center symmetric. In accord with the conclusions of [21], though, the presence of a center symmetry is really secondary for what follows.

For small A_0, and static $A_i \neq 0$, this reduces to the expected form, $E_i = D_i A_0$, as in (2). This rules out using an E_i constructed entirely from the eigenvalues of

L [19, 20]. The simplest example is $E_i \sim \partial_i \lambda$, with an infinity of other terms, such as $|\mathrm{tr} L|^2$ times this, *etc.*

There is one last limit which is essential in establishing (5), although its origin will only be clear after the discussion of interfaces below. I require that when $A_i = 0$, and A_0 is static and diagonal — but of *arbitrary* magnitude — that it reduces to the abelian form, $E_i = \partial_i A_0$. This forbids an infinity of terms, formed by taking various combinations of traces of L times (5), such as $|\mathrm{tr} L|^2$, *etc.* (Equivalently, one can write these terms times (8), as in (9)-(13) of [24].) To leading order, these conditions uniquely determine E_i. In mathematics, (5) is known as the left invariant one form of L [25].

Using the properties of path ordering, the effective electric field can be written as

$$E_i(x)/T \;=\; \int_0^{1/T} d\tau \; L(x,\tau)^\dagger \; \partial_i A_0(x,\tau) \; L(x,\tau) \\ - \; L(x,1/T)^\dagger \left[A_i(x), L(x,1/T) \right] \; . \tag{6}$$

Up to the various Wilson lines — which are, after all, phase factors in the gauge group — this is a plausible form for a gauge covariant electric field formed by averaging over τ.

With this E_i, the effective Lagrangian is that of a gauged, nonlinear sigma model [3, 26]:

$$\mathcal{L}^{eff}_{classical}(A_i, L) = \frac{1}{2}\,\mathrm{tr}\, G^2_{ij} + \frac{T^2}{g^2}\;\mathrm{tr}\;\left| L^\dagger D_i L \right|^2 \; . \tag{7}$$

Using the decomposition of the Wilson line in (4), the electric field term is proportional to

$$\mathrm{tr}\;|D_i L|^2 \;=\; \mathrm{tr}\;(\partial_i \lambda)^2 \;+\; \mathrm{tr}\;\left| \left[\Omega\, D_i\, \Omega^\dagger, e^{i\lambda} \right] \right|^2 \; . \tag{8}$$

The first term is that of an abelian theory, while the second couples the nonabelian electric and magnetic sectors together. Since $e^{i\lambda}$ is invariant under static gauge transformations, so is the combination $\Omega\, D_i\, \Omega^\dagger$. ($\Omega D_i \Omega^\dagger$ does transform under the hidden $U(1)^r$ symmetry, although the Lagrangian is invariant.)

On the lattice, the analogy of (8) is well known from Banks and Ukawa [27]. I suggested (8) previously [18], but only by expanding in small A_0. This does not suffice to fix its form at large A_0. For a related linear model, see [28].

The effective Lagrangian of (7) is not renormalizable in three dimensions (it is in two [24]), but this is a standard feature of effective theories [5]. It is also common that the effective fields are only indirectly related to those in the original theory, although it is especially striking here.

As an aside, I remark that the instanton number in four dimensions carries over directly to the effective theory. Start with a smooth, strictly periodic classical field, $A_\mu(x,\tau)$, and then transform to $A_0 = 0$ gauge. The gauge transformation which does this is just $L(x,\tau)$, (2). The instanton number is then a difference of Chern-Simons terms between $\tau = 1/T$ and 0 [15]. One can show that the instanton number equals the winding number of the Wilson line:

$$\frac{1}{24\pi^2}\;\int d^3x\, \epsilon^{ijk}\,\mathrm{tr}\,(B_i B_j B_k) \quad ; \quad B_i = L^\dagger \partial_i L \; . \tag{9}$$

Like the instanton number, this is an integer.

To establish (7), it is necessary to show that it gives the same physics as the original theory, especially at large A_0. One possibility is to use the interfaces which exist because the λ_a's are periodic.

This is most familiar for the $Z(N)$ interface of a $SU(N)$ gauge theory without quarks [17, 28]. This is given by taking a box, long in one spatial direction, with $L = 1_N$ at one end of the box, and $L = e^{2\pi i/N} 1_N$ at the other. A $Z(N)$ interface is related to the disorder parameter of 't Hooft [29, 30].

The interface which corresponds to $\mathcal{U}_c$, (3), is given by taking $A_0 = 0$ at one end of the box, and $A_0 = 2\pi T t_N/g$ at the other [31]. This is a $U(1)$ interface: while $L = 1_N$ at both ends of the box, the change in A_0 is nontrivial, and cannot be undone. Without quarks, there is a row of N distinct $Z(N)$ interfaces; with quarks, these coalesce into one $U(1)$ interface. The $U(1)$ interface is not related to a disorder parameter, since like the expectation value of L, it exists in both phases. Because they persist in the low temperature phase, though, $U(1)$ interfaces might be of cosmological interest, by forming domain walls in the early universe.

To leading order in g^2, it is easy to use interfaces to match the effective theory to the original. In the original theory, compute for constant L to one loop order. For a $SU(N)$ gauge theory without quarks, this gives [15]

$$\mathcal{L}^{eff}_{1\;loop}(L) = -\,\frac{2\,T^4}{\pi^2}\;\sum_{m=1}^{\infty}\;\frac{1}{m^4}\;\left|\mathrm{tr}\,L^m\right|^2 + \frac{\pi^2 T^4}{45} \; . \tag{10}$$

To leading order, the effective Lagrangian is the sum of (7) and (10) [17]. Because $E_i = \partial_i A_0$ when $A_i = 0$, and A_0 is static, diagonal, and of arbitrary magnitude, trivially a $Z(N)$ interface is the same in both theories. Dynamical quarks add new terms to the potential, which lift the $Z(N)$ symmetry, and so remove $Z(N)$ interfaces. $U(1)$ interfaces remain, and are analyzed similarly, with the same result for E_i.

At higher order, matching between the original and effective theories is much more involved. The effective Lagrangian is constructed from L and G_{ij} in a derivative expansion, with terms for constant L [4, 11, 12], two derivatives [11, 17, 18, 19, 20, 24], four [19], and so on. At higher order, matching will involve computing both the interface tension, $\sim T^2/\sqrt{\alpha_s}$, and expectation values of gauge invariant operators in the presence of an interface.

Further, my discussion of (2) was incomplete: it is merely the first step of three, with the others integrating out the electric and magnetic sectors [5, 6, 7, 8, 9]. While A_0 is large at the center of an interface, it is small at the ends, and so there the electric sector will have to be treated more carefully. For the pressure, it should be possible to isolate that piece which is L dependent, after subtracting the vacuum energy of the static magnetic sector for $L = 0$ [9].

While meaningful statements can only be made after computation at next to leading order, when the scale of α_s is set, qualitatively much of the physics can be understood from (10). The perturbative vacuum, $\langle L \rangle = 1_N$, gives minus the pressure of an ideal $SU(N)$ gas, $p_{ideal} = -\mathcal{L}^{eff}_{1\ loop}(1_N) = +(N^2-1)\pi^2 T^4/45$. This is the absolute minimum at leading order, and it is at least metastable, order by order in α_s.

In a $SU(N)$ gauge theory without quarks, deconfinement is related to the breaking of a global $Z(N)$ symmetry: under a $Z(N)$ transformation, $L \to zL$, where $z = e^{2\pi i/N}$. Consider the diagonal $SU(N)$ matrix

$$L_c = \text{diag}\left(1, z, z^2 \ldots z^{N-1}\right) \tag{11}$$

For L_c, a $Z(N)$ transformation just permutes the elements. Since a trace is invariant under permutations, $Z(N)$ charged loops automatically vanish: $\text{tr}\,(L_c)^{mn} = 0$ when $m = 1 \ldots (N-1)$, and n is any integer. Since $z^N = 1$, $L_c^N = 1_N$, and $Z(N)$ neutrals loops do not vanish: $\text{tr}\,(L_c)^{nN} = N$. These are precisely the properties expected for the $Z(N)$ symmetric, confined vacuum [16, 32, 33]. However, at leading order, (10), $p(L_c) = -\mathcal{L}^{eff}_{1\ loop}(L_c) = -(1 - 1/N^2)\pi^2 T^4/45$. Thus for any finite N, L_c has negative pressure, and is not a physical state.

At infinite N, however, L_c *does* represent the confined vacuum. It is $Z(\infty)$ symmetric, with $\text{tr}\,L^n = 0$ for all n. While pressure is negative, this is ~ 1, and is negligible relative to that $\sim N^2$ in the deconfined phase [4, 32, 33]. While (10) is only valid at leading order, since any trace of L_c vanishes at $N = \infty$, the pressure for L_c remains ~ 1 to all orders in $\alpha_s N$ [33].

At infinite N, L_c is familiar from random matrix models: there is complete eigenvalue repulsion, and a flat eigenvalue density [3, 4, 32]. Numerical simulations suggest that in the confined phase, the eigenvalue density for small N is like that of $N = \infty$. By factorization, in the confined phase the expectation value of the renormalized adjoint loop is $\sim 1/N^2$ [11]. For $N = 3$, though, numerically this is found to be not $\sim 10\%$, but only $\sim 1\%$ [11, 13]. That the expectation value of a $Z(N)$ neutral loop is so small indicates that the functional integral is close to an integral over the group measure; *i.e.*, that the eigenvalue density is nearly flat.

In perturbation theory, though, there is no sign of any eigenvalue repulsion which might produce a flat distribution. As in (10), and seen to three loop order in [4], the perturbative potential for constant L only involves sums of eigenvalues, and not differences. Thus eigenvalue repulsion, and so confinement, must be generated by fluctuations in the effective theory.

It is known how this happens for $SU(\infty)$ on a very small sphere [4]. The effective Lagrangian is a single integral for the constant mode of L: as a random matrix model, the Vandermonde determinant in the measure generates eigenvalue repulsion and drives the transition [3, 4, 12]. In infinite volume, though, terms in the measure depend upon the regularization; *e.g.*, they vanish with dimensional regularization.

To represent the non-perturbative effects which drive the transition in infinite volume, and motivated in part by numerical simulations [34, 35], by hand I add to the effective Lagrangian

$$\mathcal{L}^{eff}_{non-pert.} \sim +B_f\, T^2\, |\text{tr}\, L|^2 \ . \tag{12}$$

This is the simplest term of many, such as $|\text{tr}\,L^2|^2$, *etc.*. The constant B_f has dimensions of mass squared, and so is a pure number times T_c^2 [36].

After adding such a term to the effective Lagrangian, the minimum of the loop potential is for some $\langle L \rangle \neq 1_N$, which produces a Higgs effect. Because L is an adjoint field, in perturbation theory the mass magnetic gluons acquire from this Higgs effect involves differences of eigenvalues: diagonal gluons remain massless, while off diagonal gluons develop a mass [23]. Integrating out fluctuations in A_i and L to one loop order (which is easiest in unitary gauge), there is a qualitatively new term in the effective Lagrangian,

$$\Delta\mathcal{L}^{eff} \sim -\sum_{a,b=1}^{N} \left(g^2 |e^{i\lambda_a} - e^{i\lambda_b}|^2\right)^{3/2} \ . \tag{13}$$

The sign is physical, and corresponds to eigenvalue repulsion.

These calculations are only suggestive. For an adjoint field, it is not obvious how to characterize a Higgs effect gauge invariantly [37], but this is secondary. The effective theory, as determined perturbatively, can be studied in various ways. Since the ultraviolet cutoff is physical, it is reasonable to start with mean field theory [11, 12]. To do better, the theory could be simulated numerically, to directly measure the eigenvalue distribution. At large N [35], analytic approximations may help [38].

The usual justification for using an effective Lagrangian is the presence of a small mass scale, but generically, there is none here. If the effective coupling is small at T_c [7], though, then with care nothing is lost by going to an effective Lagrangian. Presumably, this is worthwhile when (2) fails: from $\sim 3T_c$, down to some point below T_c [39]. For constant L, the effective potential shows no signs of a transition to a confining phase: at $N = \infty$, and perhaps even for small N, this must involve eigenvalue repulsion. In infinite volume, this arises dynamically, especially from fluctuations in the angular variables, Ω, and the gauge fields, A_i. This, then, is why the effective theory is of interest: we can use it to uniquely isolate the dynamic origin of the transition, as eigenvalue repulsion. It thus provides a notable example of a field theory of (not so) random matrices [3].

Acknowledgements: This research was supported by D.O.E. grant DE-AC02-98CH10886, and in part by the Alexander von Humboldt Foundation. I most gratefully acknowledge discussions with D. Diakonov. During a sabbatical which I took at the Niels Bohr Institute in '03-'04, he stressed to me that the usual effective Lagrangians are not center symmetric. I also thank D. Boer, J. Berges, M. Creutz, O. Kaczmarek, F. Karsch, C. Korthals-Altes, D. Kharzeev, M. Laine, and V. P. Nair for their comments.

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[36] Using the data of [34], one can show that in a pure $SU(3)$ gauge theory, that if e is the energy density and p the pressure, that $(e-3p)/T^2$ is nearly constant from $\sim 1.1\,T_c$ to $4.0\,T_c$. (Because the $N=3$ transition is nearly second order, this quantity rises sharply from near zero at T_c, to a maximum at $\sim 1.1\,T_c$.) This, and $p(T_c) \approx 0$, implies that to a good approximation,

$$p(T) = a\left(T^4 - T_c^2\, T^2 \right) \quad , \quad T > 1.1\, T_c \; .$$

Fitting to $p(4T_c)$, the constant $a \approx 0.9$ of the ideal gas value. In a MIT bag model with bag constant B, $(e -$

$3p)/T^2 \sim B/T^2$. Thus one can view the $\sim T^2$ term in the pressure as a temperature dependent, or "fuzzy", bag constant, $\sim B_f$. Even so, this form for the pressure only suggests (12) and does not explain it (or vice versa).

[37] While diagonal and off-diagonal gluons cannot be easily separated, a Higgs effect does mix magnetic glueballs with the Wilson line. To see this, it may be useful to measure the masses of magnetic glueballs not only from ordinary spatial plaquettes, but from spatial plaquettes split in τ: take two spatial links at $\tau = 0$, and two at $\tau = 1/T$, tied together by two thermal Wilson lines.

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